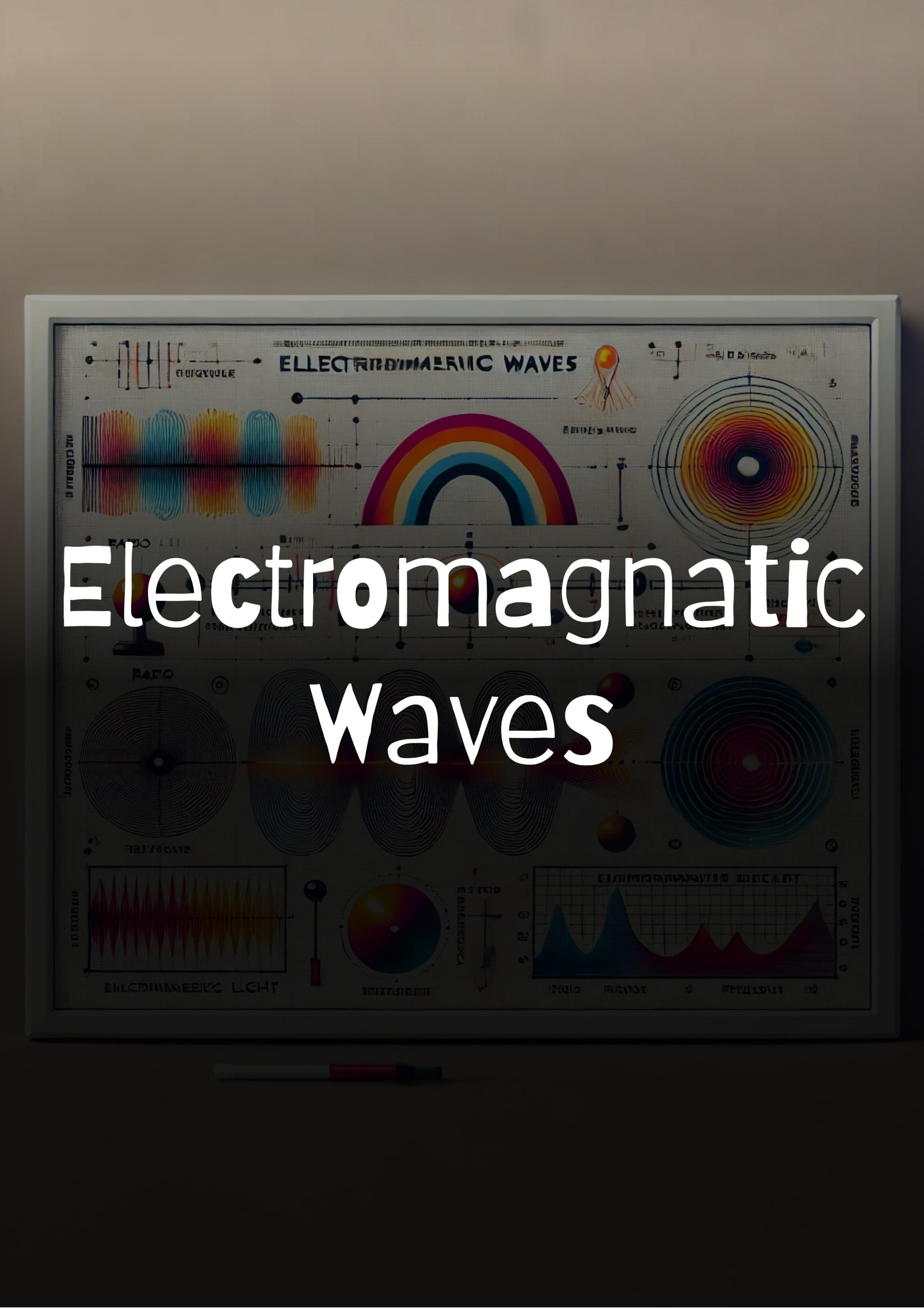
Electromagnatic Waves



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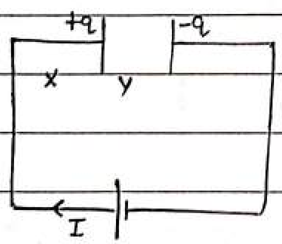
ELECTROMAGNETIC WAVE

Electrostatic ⟶ Electric field
M.E.C | Magnetism ⟶ Magnetic field
EMI. ⟶ Time varying magnetic field produce electric field
EMW ⟶ Time varying electric field will induce magnetic field.

Charge	Electric field	Magnetic field	EM wave
Rest	✓	×	×
V = constant	✓	✓	×
Acceleration	✓	✓	✓

Charging of Capacitor

↳ where is symmetricity of current



at x	at y
According to Ampere circuital law	According to Ampere circuital law
$B_x \neq 0$	$B_y = 0$
Maxwell experimentally find $B_x \neq 0$	Maxwell experimentally find $B_y \neq 0$

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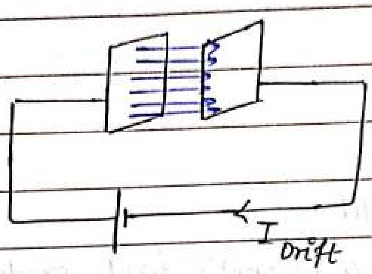
Shivalik

Displacement current



Displacement current is the current between the capacitor plate during charging of capacitor and that displacement current is responsible for magnetic field b/w the capacitor plate.

$$I_D = \epsilon_0 \frac{d\phi_E}{dt}$$ → electrostatic flux

Q What is the reason of displacement current & prove that

$I_D = \epsilon_0 \frac{d\phi_E}{dt} = ?$

Ans Varying electric flux (Due to varying charge on capacitor) between the capacitor plate is a reason of displacement current.

$\phi = E.A$

$\phi = \frac{\sigma}{\epsilon_0} A = \frac{q}{A \epsilon_0} A$

$q = \phi \epsilon_0$

$\frac{dq}{dt} = \frac{d\phi \epsilon_0}{dt}$

$\frac{dq}{dt} = I_D = \epsilon_0 \frac{d\phi}{dt}$

$$I_D = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 A \frac{dE}{dt} = \frac{\epsilon_0 A}{l} \frac{dv}{dt}$$

The MR*

$q = cv$ [Charge at time t]

$$\frac{dq}{dt} = c\frac{dv}{dt}$$

$$I_D = C\frac{dv}{dt} = \frac{\epsilon_0 A}{l}\frac{dv}{dt} = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 \frac{d\phi_E}{dt}$$

DISPLACEMENT CURRENT

Formed due to changing electric field

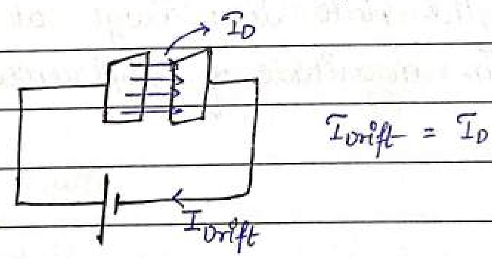
Proves continuity of current.

Explains existence of magnetic field b/w the magnetic field capacitor plates

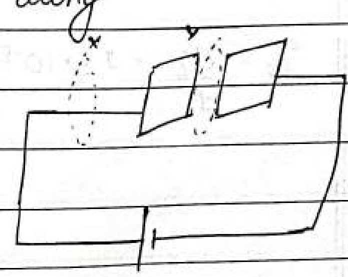
It does not exist under steady state.

$$I_D = \epsilon_0 \frac{d\phi_E}{dt}$$

Flows b/w the cross section area of capacitor plate.



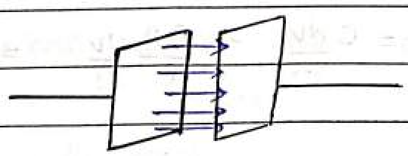
Q A parallel plate capacitor is charged by a battery as shown in the figure. If two circular amperian loops x and y are drawn then $\oint \vec{B} \cdot \vec{dl}$ will be zero along



(A) x only
(B) Y only
(C) Both x and y

d) neither x nor y

Q A parallel plate capacitor with plate area A and separation between them is d, is charged by a current I. Displacement current through a plane surface of area A/2 parallel to the plates and drawn symmetrically between the plates is.



a) I
b) I/4
c) I/2
d) 3I/2

ANS $I_D = \epsilon_0 A \frac{dE}{dt}$

$I_D = \epsilon_0 A \frac{dE}{dt}$ → $I_2 = \frac{I_1}{2} = \frac{I}{2}$

$I_D \propto A$

Q A parallel plate capacitor is charged to 60 μC. Due to a radioactive source, the plate loses charge at the rate of 1.8×10^{-8} Cs^{-1}. The magnitude of displacement current is

a) 3.6×10^{-8} Cs^{-1}
b) 1.8×10^{-8} Cs^{-1}
c) 4.1×10^{-11} Cs^{-1}
d) 5.7×10^{-12} Cs^{-1}

ANS $I_D = \frac{dq}{dt} = 1.8 \times 10^{-8}$ Cs^{-1}

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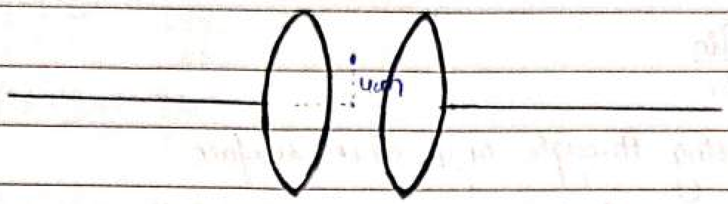
Q Two circular plates of radius 0.1 m are used to form a parallel plate capacitor. If displacement current between the plates is 2π ampere, then find magnetic field produced by displacement current 4cm from the axis of the plates.



Ans $I \propto A \Rightarrow I \propto r^2$

$$I' = \frac{I r^2}{R^2} = \frac{I \times 16}{100}$$

$$B = ?$$

$$\oint B \cdot dl = I_{in} \mu_0$$

$$B \times 2\pi \times 4cm = \frac{I \times 16}{100} \mu_0$$

$$B \times 2\pi \times 4 \times 10^{-2} = \frac{2\pi \times 16 \mu_0}{100} = 4\mu_0 \text{ Ans}$$

Q A parallel plate capacitor consists of two circular plates of radius 0.05 m. If electric field b/w the plates is changed as $\frac{dE}{dt} = 10^{10} \frac{V}{m\text{-}s}$ then find displacement current between the plates.

Ans $$I_D = \epsilon_0 A \frac{dE}{dt} = (4\pi \times 9 \times 10^9)^{-1} \times \pi \times 25 \times 10^{-4} \times 10^{10}$$

$$\Rightarrow \frac{1}{4\pi \times 9 \times 10^9} \pi \times 25 \times 10^{-4} \times 10^{10} = \frac{25 \times 10^{-3}}{36} = 0.0007 \text{ A Ans}$$

FOUR MAXWELL'S EQUATION

1) Gauss law of electrostatic

$$\phi = \oint \vec{E} \cdot \vec{dA} = \frac{q_{in}}{\epsilon_0}$$

total flux passing through any close surface is $\frac{1}{\epsilon_0}$ of charge enclose.

2) Gauss law in magnetism

$$\oint \vec{B} \cdot \vec{ds} = 0$$

Monopole does not exist magnetic field lines always form a close loop.

3) Induced Electric field

$$emf = -\frac{d\phi_B}{dt}$$

$$\oint E \cdot dl = \mp \frac{d\phi_B}{dt}$$

4) Ampere-Maxwell law.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 [I_{Drift} + I_{Displacement}]$$

Four Maxwell equation → $\vec{E}$ & $\vec{B}$ ka close integral

Area | length

1) $\oint \vec{E}\cdot\vec{dl} = -\frac{d\phi_B}{dt}$

2) $\oint \vec{E}\cdot\vec{dA} = \frac{q_{in}}{\epsilon_0}$

3) $\oint \vec{B}\cdot\vec{dl} = \mu_0 [I_{Drift} + I_{Displacement}]$

4) $\oint \vec{B}\cdot\vec{dA} = 0$

Faraday's law of EMI.

1) Time varying magnetic field produced electric field

2) Both are perpendicular to each other and induced electric field is proportional to rate of change in magnetic field.

$$\oint \vec{E}\cdot\vec{dl} = \frac{d\phi}{dt} = A\frac{dB}{dt}$$

Varying electric field produced magnetic field; unification of electric and magnetic field.

$$\oint \vec{B}_{in}\cdot\vec{dl} = \mu_0 \left[I_{Drift} + \epsilon_0 A \frac{dE}{dt} \right]$$

Source of em wave

(i) Accelerating charge

(ii) L-C oscillation.

3) Transition of e^o in orbit
4) Retardation of e^o when it enters into a target of higher atomic weight
5) De-excitation of e^o in radioactivity.

Electromagnetic wave

→ Energy transfer due to oscillation of electric and magnetic
→ field.

→ Non mechanical wave
→ Does not require medium
→ Transverse wave
* → $q_{net} = 0$
→ hence EM wave does not deviate in external electric and magnetic field.

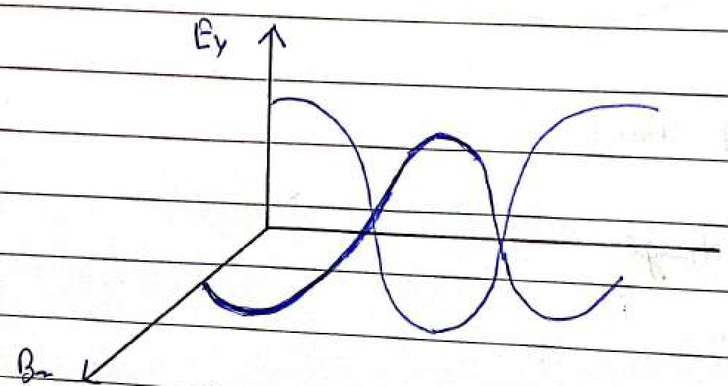
Production of EM wave in L.C oscillation.

$q = q_o \cos(\omega t)$

$\frac{q}{A\epsilon_o} = E = \frac{q_o}{A\epsilon_o}\cos(\omega t) = E_o \cos(\omega t)$

↓

$E = E_o \cos(\omega t \pm kx)$



Electric and magnetic fields are ⊥r to each other.
Wave is moving ⊥r to E and B that it is in the direction of $\vec{E} \times \vec{B}$

$\vec{E}$ & $\vec{B}$ are in same phase

E_0 = peak value of electric field

$B_0 = \frac{E_0}{c}$ = peak value of magnetic field

C = speed of light in vacuum

V = speed of light in medium

$V = \frac{C}{\text{Refractive index}}$

$B_0 = \frac{E_0}{V}$

$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ $\quad V = \frac{1}{\sqrt{\mu_m \epsilon_m}}$

$E_y = E_0 \sin(\omega t - kx)$
↳ wave is in +x direction

$B_z = B_0 \sin(\omega t - kx)$

ω = Angular frequency $= \frac{2\pi}{T} = 2\pi f$

$k = \frac{2\pi}{\lambda}$ = Angular wave no.

$k = \frac{1}{\lambda}$ = wave no.

$V = \frac{E_0}{B_0} = \frac{\omega}{k} = \frac{\lambda}{T}$

Nature of electromagnetic wave

1) Speed of E-M wave $= 3\times10^8\ m/s = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

2) Speed of Em wave in medium $= \frac{1}{\sqrt{\mu_m \epsilon_m}}$

3) Electric and magnetic field oscillate in same phase

4) Direction of Em wave $= \vec{E}\times\vec{B}$

5) Energy of Em wave equally divided in electric and magnetic field

6) (Energy density) of Em wave $= \epsilon_0 E^2 = \frac{B^2}{\mu_0}$

γ-ray   Xray   UV rays   {Visible light}   I.R.   MW   RW.

←

$f\uparrow\ \lambda\downarrow\ E\uparrow$

$E = h\nu = \frac{hc}{\lambda}$

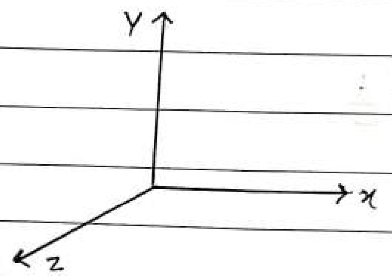
Q Light wave travelling along y direction. If the corresponding $\vec{E}$ at any time is along x axis, the direction of $\vec{B}$ vector at that time is along.

a) y axis

b) x axis

c) z axis

d) -z axis



ANS $\vec{E} \times \vec{B}$ = Direction of wave propagation

$\hat{i} \times (-\hat{k}) = \hat{j}$

Q The electric and magnetic field of an electromagnetic wave are in .... phase and ...... to each other

a) same, parallel
b) opposite, perpendicular
c) opposite, parallel
d) same, perpendicular.

Q An em wave is propagating in a medium with a velocity $\vec{v} = v\hat{i}$. The instantaneous oscillating electric field of this em wave is along +y axis then the direction of oscillating magnetic field of the em wave will be along.

a) -z direction
b) +z direction
c) +x direction
d) -y direction

direction of em wave = 1st vector × 2 vector (Map.)

Energy	1 MeV	1 KeV	10eV-100eV	1ev to 2eV	1meV	1 μeV	1neV
	γ ray	X ray	U.V.	V.L	I.R	M.W.	R.W.
	10^{20} Hz	10^{18} Hz	10^{16} Hz	10^{15} Hz	10^{12} Hz	10^{8} Hz	10^{4} Hz

$f \uparrow \quad \uparrow E = hf \uparrow$

$\lambda = \frac{c}{f}$ $\quad E = \frac{hc}{\lambda}$ $\quad \lambda = \frac{hc}{E}$ → $E = \frac{12400\,eV}{\lambda(\text{Å})}$

$E = \frac{hc}{\lambda} = \frac{hcP}{h}$ $\quad [\because \lambda = \frac{h}{mv} = \frac{h}{P}]$

$E = Pc$

$P = \frac{E}{c}$

Q Frequency of the wave is 6×10^{15} Hz. The wave is.

a) Radiowave
b) Microwave
~~c)~~ X ray → most appropriate
c) Infrared

Q The energy of the em wave is of the order of 15 keV. To which part of the spectrum does it belong?

a) γ rays
~~b)~~ X rays
c) Infrared rays
~~d)~~ Ultraviolet rays.

Q The magnetic field in a plane electromagnetic is given by = $2\times10^{-7}\sin(0.5\times10^{3}x + 1.5\times10^{11}t)$
This electromagnetic wave is.

a) Visible light
~~b)~~ Infrared
c) Microwave
d) Radio wave

ANS $\omega = 2\pi f = 1.5\times10^{11}$
$2\times\pi f = \frac{3}{2}\times10^{11}$
$f \simeq 2.5\times10^{10}$

Q λ_v, λ_x and λ_m represent the wavelengths of visible light, x rays and microwaves respectively then.

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a) $\lambda_m > \lambda_x > \lambda_v$

b) $\lambda_v > \lambda_m > \lambda_x$

c) $\lambda_m > \lambda_v > \lambda_x$

d) $\lambda_v > \lambda_x > \lambda_m$

Q The oscillating magnetic field in a plane electromagnetic wave is given as.

$$B_y = 8\times10^{-6} \sin(5000\pi x - 3\times10^{11}\pi t)\ T.$$

a) Calculate

Frequency

$\Rightarrow 3\times10^{11}\pi = 2\pi f$

$f = 1.5\times10^{11}$

b) Wavelength.

$5000\pi = \frac{2\pi}{\lambda}$

$\lambda = 4\times10^{-4}\ m$

c) Speed of the wave

$v = f\lambda = 6\times10^{7}$

d) Electric field amplitude

$E_0 = \frac{B_0}{v} = \frac{8\times10^{-6}}{6\times10}$

$E_0 = B_0 v = 48\times10^{7}\times10^{-6}$

$= 480$

e) Write down the expression for oscillating electric field.

$E_z = E_0 \sin(5000\pi x - 3\times10^{11}\pi t)$

Q Which of the following is not transported by electromagnetic wave?

a) Energy

b) Momentum

c) Charge

d) Information

Q The speed of EM waves depends upon.

a) Wavelength
b) Frequency
c) Intensity
d) Medium, in which it travels.

Q Velocity of EM wave in a medium is

a) $(\epsilon_0 \mu_0)^{-1/2}$
b) $(\epsilon_0 \epsilon_r \mu_0 \mu_r)^{-1/2}$
c) 3×10^8 m/s
d) $(\epsilon_0 \epsilon_r / \mu_0 \mu_r)^{+1/2}$

Ans $V = \frac{1}{\sqrt{\mu_m \epsilon_m}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}}$

Q An electromagnetic wave is propagating in vacuum along z-axis, the electric field component is given by $E_x = E_0 \sin(kz - \omega t)$ then magnetic component is

a) $B_x = \frac{E_0}{c} \sin(kz - \omega t)$

b) $B_y = \frac{B_0}{c} \sin(kz - \omega t)$

c) $B_y = \frac{E_0}{c} \sin(kz - \omega t)$

d) $B_y = B_0 c \sin(kz - \omega t)$

Q The speed of electromagnetic waves in a medium (whose dielectric constant is 2.25 and relative permeability is 4) is equal to

a) 0.5×10^8 m/s
b) 0.25×10^8 m/s
c) 0.75×10^8 m/s
d) 1×10^8 m/s

Ans $V = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{3 \times 10^8}{\sqrt{9}} = 10^8$

Q If an electromagnetic wave propagating through vacuum is described by $E_y = E_0 \sin(kx - \omega t)$; $B_z = B_0 \sin(kx - \omega t)$ then

a) $E_0 k = B_0 \omega$ $\qquad V = \frac{E_0}{B_0} = \frac{\omega}{k}$
b) $E_0 B_0 = \omega k$
c) $E_0 \omega = B_0 k$
d) $E_0 B_0 = \omega / k$

Q Red light differs from the blue light in.

a) Speed
b) frequency
c) Intensity
d) Amplitude

Q Out of the following, choose the ray which does not travel with the velocity of light

a) X ray
b) Microwave
c) γ rays
d) β-rays.
↓
Beam of fast moving electrons

ENERGY DENSITY

Electrostatic energy density in EM wave $= \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 E_0^2 \sin^2(\omega t - kx)$ (instantaneous)

Average electrostatic energy density in EM wave $= \langle u_E \rangle = \frac{1}{2}\epsilon_0 E_0^2 \langle \sin^2(\omega t - kx) \rangle$

$\Rightarrow \frac{1}{2}\epsilon_0 E_0^2 \times \frac{1}{2}$

$\Rightarrow \frac{1}{4}\epsilon_0 E_0^2$

Magnetic energy density in EM wave $= \frac{1}{2}\frac{B^2}{\mu_0} = \frac{B_0^2 \sin^2(kx - \omega t)}{2\mu_0}$

Average magnetic energy density in EM wave $= \langle u_B \rangle = \frac{B_0^2}{2\mu_0} \times \frac{1}{2}$

$\Rightarrow \frac{B_0^2}{4\mu_0}$

Prove that electrostatic and magnetic energy density equally divided in EM wave.

$u_E = u_B$

$u_E = \frac{1}{2}\epsilon_0 E^2$

$= \frac{1}{2}\epsilon_0 B^2 c^2$

$= \frac{1}{2}\epsilon_0 B^2 \frac{1}{\mu_0 \epsilon_0} = \frac{B^2}{2\mu_0}$

Energy density of EM wave = $u_E + u_B = 2u_E = 2u_B$

$= \frac{1}{2} \times 2\, \epsilon_0 E^2 = 2 \times \frac{1}{2} \frac{B^2}{\mu_0}$

$$u_{EM\,wave} = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

Average energy density of EM wave $= \langle u_{EM} \rangle = \epsilon_0 \langle E^2 \rangle$

$= \epsilon_0 E_0^2 \langle \sin^2(\omega t - kx) \rangle$

$= \frac{1}{2} \epsilon_0 E_0^2 = \frac{1}{2} \frac{B_0^2}{\mu_0}$

MR*

$u_E = \frac{1}{2} \epsilon_0 E^2$	$\langle u_E \rangle = \frac{1}{4} \epsilon_0 E_0^2$
$u_m = \frac{1}{2} \frac{B^2}{\mu_0}$	$\langle u_m \rangle = \frac{1}{4} \frac{B_0^2}{\mu_0}$
$u_{EM} = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$	$\langle u_{EM} \rangle = \frac{1}{2} \frac{B_0^2}{\mu_0} = \frac{1}{2} \epsilon_0 E_0^2$

Q In a plane electromagnetic wave, which of the following has / have zero average value in complete cycle?

a) Magnetic field (b) Magnetic energy

c) Electric field d) Electric energy

~~a)~~ a, c

b) b, c

c) a, d

d) All of these

Q In an electromagnetic wave in free space the root mean square value of the electric field is $E_{rms} = 6\,V/m$. The peak value of the magnetic field is.

a) $1.41 \times 10^{-8} T$

~~b)~~ $2.83 \times 10^{-8} T$

c) $0.70 \times 10^{-8} T$

d) $4.23 \times 10^{-8} T$

Ans $E_{rms} = B_{rms} C$

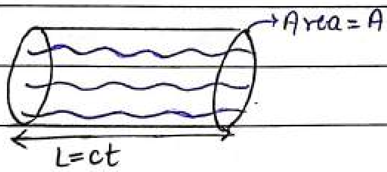
$$\frac{6}{c} = B_{rms} = \frac{B_0}{\sqrt{2}}$$

$$B_0 = \frac{2 \cdot 6\sqrt{2}}{3 \times 10^8} = 2.8 \times 10^{-8} T \text{ Ans}$$

INTENSITY OF EM WAVE

Energy of EM wave crossing per unit area per second in a direction perpendicular to the direction of propagation.

$$I = \frac{E}{At}$$



Average energy density of EM wave $= \frac{1}{2}\epsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0} = \frac{E}{\text{Volume}}$

Energy of EM wave $= \frac{1}{2}\epsilon_0 E_0^2 \text{ volume} = \frac{1}{2}\epsilon_0 E_0^2 ctA$

$$I = \frac{\text{Energy}}{At} = \frac{\frac{1}{2}\epsilon_0 E_0^2 ctA}{At} = \frac{1}{2}\epsilon_0 E_0^2 c$$

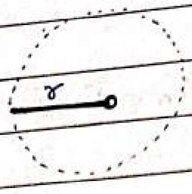
$$I = \frac{1}{2}\epsilon_0 E_0^2 c$$

$$I = \frac{1}{2}\frac{B_0^2 c}{\mu_0}$$

Intensity due to a point source

The power of source = P

Distance of point A where intensity to be calculated. = r



For line source

$I = \frac{E}{A \times t}$

$I = \frac{E}{2\pi r l t}$

$I = \frac{E}{4\pi r^2 t}$

$I \propto \frac{1}{r}$

$I \propto \frac{1}{r^2}$

$$I = \frac{E}{At} = \frac{P}{A}$$

Relation b/w P and ~~time~~. and intensity.

$P = \frac{E}{t}$

$I = \frac{E}{At} = \frac{P}{A}$

Poynting vector or Energy flux

$\vec{S}$ = (Magnitude of intensity) × Direction of wave

$\vec{S} = |I| (\hat{E} \times \hat{B})$

Magnitude of Poynting vector = Intensity.

$|\vec{S}| = \frac{1}{2} \epsilon_0 E_0^2 c = I$

$= \frac{1}{2} \epsilon_0 E_0 E_0 c$

$= \frac{1}{2} \epsilon_0 E_0 B_0 c^2$

$= \frac{1}{2} \epsilon_0 E_0 B_0 \frac{1}{\mu_0 \epsilon_0} = \frac{1}{2} \frac{E_0 B_0}{\mu_0}$

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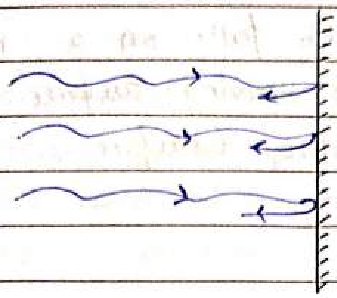
Change in momentum and Radiation pressure.



$\Delta P = \vec{P_f} - \vec{P_i}$

$\Delta P = -P - P$

$\Delta P = -2P$

$|\Delta P| = 2P = \frac{2E}{c}$

$Force = \frac{\Delta P}{\Delta t} = \frac{2E}{ct}$

$Force = \frac{2IA\cancel{t}}{c\cancel{t}}$

$Force = \frac{2IA}{c}$

Divide by A both side

$\frac{F}{A} =$ radiation Pressure $= \frac{2I}{c}$

Complete absorbing surface

$\Delta P = \vec{P_f} - \vec{P_i}$

$\Delta P = P$

$\Delta P = \frac{E}{c}$

Radiation pressure $= \frac{I}{c}$

Q The ratio of contributions made by the electric and magnetic field components to the intensity of an electromagnetic wave is (c = speed of electromagnetic waves)

a) C : 1

b) 1 : 1

c) 1:c

(d) $1:c^2$

Q Light with an average flux of 20 W/cm² falls on a non-reflective surface at normal incidence having surface area 20cm². The energy received by the surface during time span of 1 minute is.

a) 10×10^3 J

b) 12×10^3 J

c) 24×10^3 J

d) 48×10^3 J

Ans $I = \frac{E}{At} = \frac{P}{tA} = 20$ W → watt

$\frac{E}{At} = 20$

$E = 20 \times 20 \times 60$

$E = 24000 J = 24 \times 10^3 J$

Q Which of the following statement is false for the properties of electromagnetic wave?

a) These waves do not require any material medium for propogation.

b) Both electric and magnetic field vectors attain the maxima and minima at the same place & same time

c) The energy in electromagnetic waves is divided equally between electric & magnetic vectors.

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d) Both electric and magnetic field vectors are parallel to each other and perpendicular to the direction of propagation of wave.

Q A parallel plate capacitor of capacitance 20 μF is being charged by a voltage source whose potential is changing at the rate of 3 V/s. The conduction current through the connecting wires, and the displacement current through the plates of the capacitor would be respectively.

a) zero, zero
b) zero, 60 μA
c) 60 μA, 60 μA
d) 60 μA, zero

Ans $I_D = \frac{\epsilon_0 A}{d} \frac{dV}{dt} = 20\mu \times 3 = 60 \mu A$

Q An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a dielectric medium with relative permittivity $\epsilon = 4.0$. Then.

a) wavelength is doubled and frequency becomes half
b) Wavelength is halved and frequency remains unchanged
c) Wavelength and frequency both remain unchanged
d) Wavelength ~~an~~ is doubled and frequency is unchanged.